

Reflectivity Function Reconstruction Using the Frequency Deconvolution Technique¹

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Abstract—Deconvolution of ultrasonic echo signals improves resolution and quality of ultrasonic images. A frequency deconvolution algorithm depends on the Fast Fourier transform is proposed for ultrasonic data. The stability of the algorithm and the influence of the truncation effect on the deconvoluted results were investigated with respect to the duration time of reflectivity function reconstruction and the signal to noise ratio. Reliability of the separation of reconstructing the reflectivity of a biological tissue is estimated by frequency deconvolution of the echo ultrasound signals.

Keywords: ultrasonic, deconvolution, Fast Fourier transform, tissue, reflectivity function

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1. INTRODUCTION

The use of deconvolution technique for both range and later resolution enhancement in ultrasonic application has been subject to widespread investigation and is well documented in the literature [1–4]. Oppenheim [5] have defined the case of estimating the medium response or the reflectivity function from the ultrasound system response as the well-known homomorphic deconvolution, using the real cepstrum for minimum phase signals or the complex cepstrum for the most general case. Torfinn [6], compares seven methods based on the cepstrum for blind deconvolution in the estimation of the reflectivity function in biological media. Neelamani [7] proposed a regularized deconvolution technique based on Wavelet (ForWARD) which has been used by Roberto [8] for the deconvolution of ultrasonic signals. Ali [9] has been used the adaptive lattice filter algorithm to estimate the reflectivity function.

In this paper, a method for signal processing based on frequency deconvolution technique has been used to enhance the axial resolution in an ultrasonic system by suitable processing of the received output. The reflectivity function (image) reconstruction is based on an algorithm evaluating the reflected and scattered signals in the A-scans. The A-scans are modelled as the tissue response of the imaged object convoluted with the shape of the ultrasonic pulse, which is determined by the transfer function of the transducers and the excitation. The stability of the algorithm and the influence of the truncation effect on the deconvoluted results were investigated with respect to the duration

time of reconstruction of reflectivity function and the signal to noise ratio.

2. SIGNAL PROCESSING

In practice it is well accepted that, one scan line of the received pulse echo signal, $y(t)$ at time t , can be represented as [10]

$$y(t) = w(t) * x(t) + n(t), \quad (1)$$

where $x(t)$ is the medium response or the reflectivity function [11], $w(t)$ is the ultrasound system response [12], $n(t)$ is the zero-mean Gaussian noise, and $*$ represents the convolution integral given by

$$p(t) * q(t) = \int_{-\infty}^{+\infty} q(\tau)p(t-\tau)d\tau. \quad (2)$$

Deconvolution attempts to remove the effect of the input function $w(t)$ from the output $y(t)$ to achieve some close approximation to the original medium impulse response $u(t)$. The Fourier transformation (FT) of the convolution of two functions can be represented as multiplication of Fourier transforms of these functions:

$$FT(y) = FT(w * x) = FT(w)FT(x), \quad (3)$$

$$FT(x) = \frac{FT(y)}{FT(w)} = \frac{YW^*}{WW^* + \varepsilon}, \quad (4)$$

where Y and W denotes the Fourier transform of y and w , W^* is a complex conjunct of W and ε represents the regularization parameter which has a small positive number as a damping factor. The reflectivity function

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reconstruction can be obtained from the received pulse-echo signal using the formula:

$$x = FT^{-1} \left[\frac{YW^*}{WW^* + \varepsilon} \right]. \quad (5)$$

The symbols FT^{-1} denotes the inverse Fourier transform.

3. SIMULATION RESULTS

3.1. Simulations of Reflectivity Function

There are a different distributions have been describing the reflectivity function of soft tissues [13–16] but the Generalized Gaussian distribution [16] is the most appropriate in this study. The Generalized Gaussian distribution probability distribution function can be written as

$$\begin{aligned} P(y) &= a \exp[-(y/b)^\xi], \\ a &= \xi/[2b\Gamma(1/\xi)], \\ b &= \sigma \sqrt{\Gamma(1/\xi)/\Gamma(3/\xi)}, \end{aligned} \quad (6)$$

where b is the scale parameter, σ is the standard deviation ξ is the shape parameter and Γ is the Gamma function. As ξ explains the signal energy, ξ is directly connected to signal sparsity and scattered concentration. Figure 1 shows the Generalized Gaussian distribution with different values of ξ . It is clear that Fig. 1 represents an accurate description of the most general tissue structure. The value of ξ equal to 0.01 represents isolated tissue reflector while for value of ξ equal to 0.1 represent diffusive regions of tissue.

3.2 Simulation of Ultrasound System Response

The results are derived from a representative range of simulated ultrasonic data, obtained using the modeling of ultrasonic piezoelectric transducers described in reference [17]. The simulation was applied using typical data for transducer constructed with a lead zirconate titanate (PZT-5A) piezoelectric element with tungsten-epoxy [18] ($z = 19 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$) back block. The transmitter response is obtained for a 10 mm diameter pulse-echo transducer of 5 MHz center frequency [19]. The impedance of the generator is assumed to be 50 ohms resistance. Figure 2 shows the expected waveform at the transducer terminals when the device acts as a receiver; that is, in pulse-echo mode with no radiation coupling, target or other field effects. The pulse-echo response of the transducer has been convoluted with the tissue reflector sequence described in Fig. 1. An uncorrelated Gaussian noise can be simply added to the overall time domain response. Figure 3 shows the overall time domain response at the receiving terminal at a signal_to_noise level of 30 dB. The deconvolution algorithm was implemented using the FORTRAN language to estimate the reconstruction of reflectivity

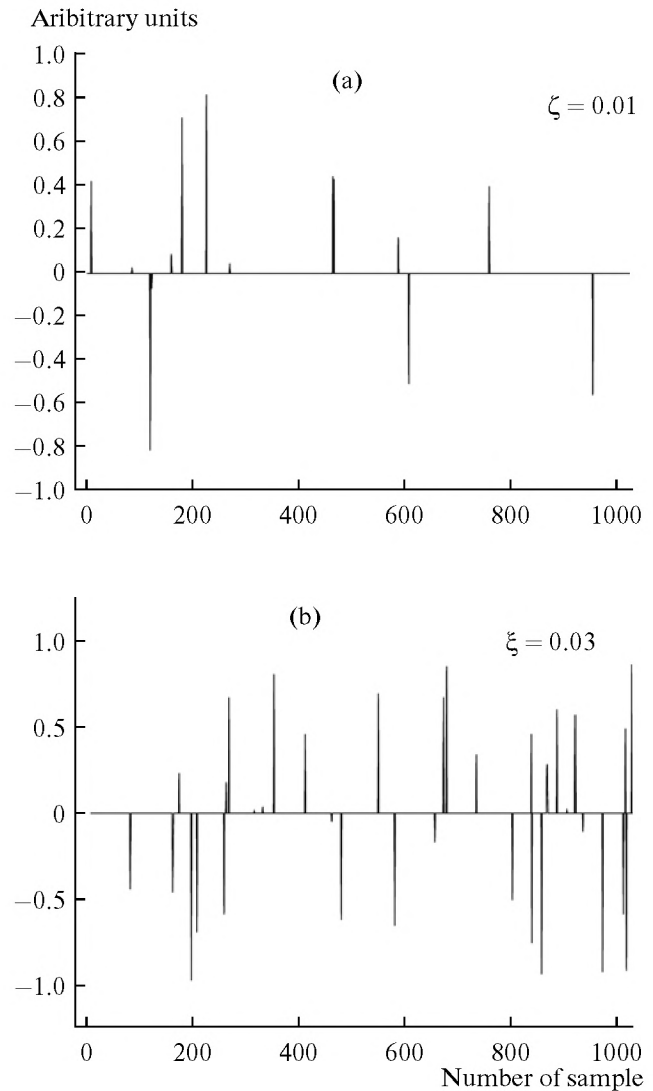


Fig. 1. The impulse reflector series of random sequences drawn from zero-mean unit variance generalized Gaussian distribution, having unit variance and shape parameter equal to (a) $\xi = 0.01$ and (b) $\xi = 0.03$.

function. It is necessary to test the algorithm with different values of ε . Figure 4 shows the variation of average spike duration time and the signal to noise ratio (dB) of reconstruction reflectivity function with different values of regularization parameter ε . From this figure, it can be seen that the suitable magnitude of ε equals to 10^{-8} . Figure 5 shows the deconvolution simulated results of reconstruction of reflectivity functions under the previously optimum value of ε . From this figure, it can be seen that the alternating sign of the reconstructed function is very close to the initial pulse. The relative Amplitudes of the responses are equal to the relative initial amplitudes, and their locations look like real locations. It is clear from these figures that the

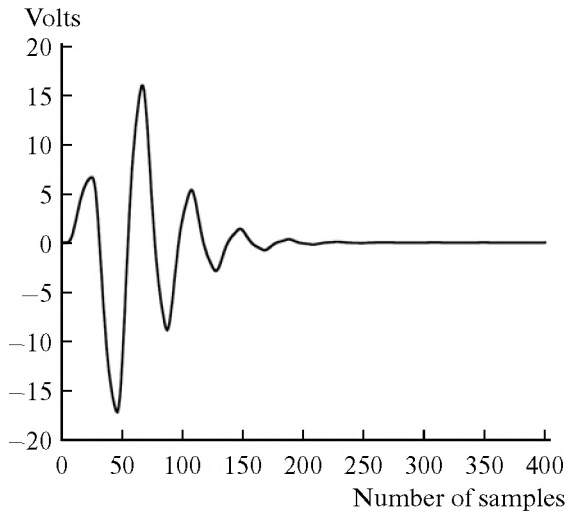


Fig. 2. The pulse-echo transducer response.

results are promising for improving resolution. The deconvolution results presented are in good agreement with the true reflectivity function.

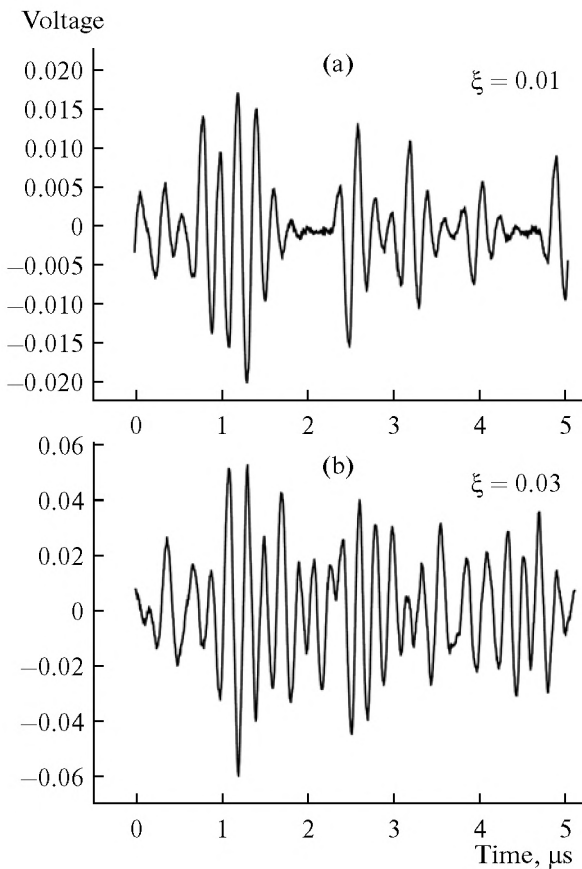


Fig. 3. The overall simulation of a pulse-echo transducer with signal to noise level 30 dB. (a) $\xi = 0.01$ and (b) $\xi = 0.03$.

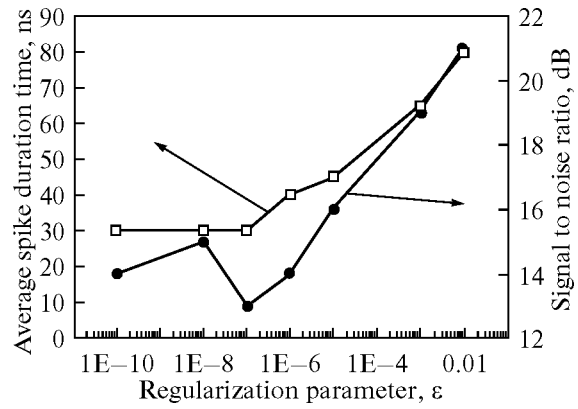


Fig. 4. Variation of reconstructing the function with signal to noise ratio and average spike duration time as a function of the regularization parameter.

4. CONCLUSION

A deconvolution algorithm to reconstruct the reflectivity function has been developed. The regular-

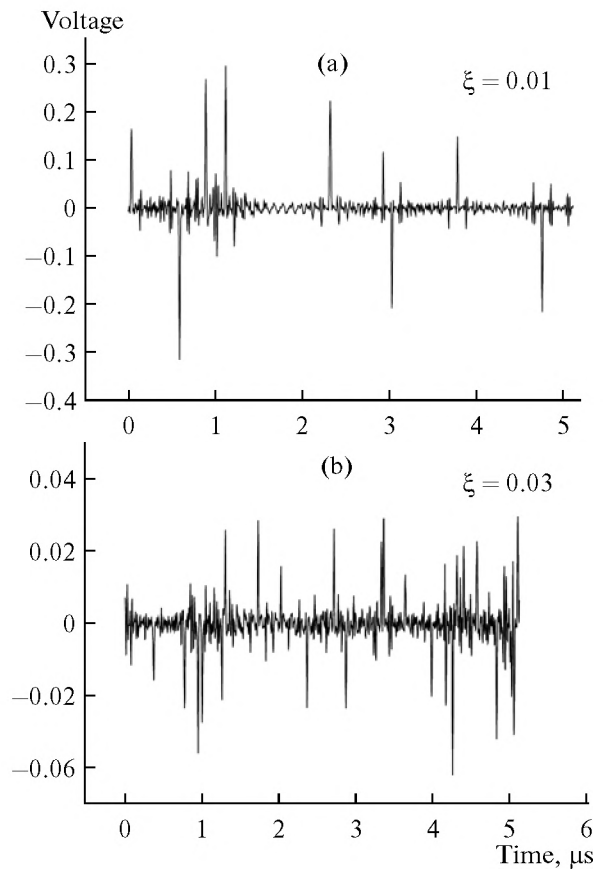


Fig. 5. Deconvolution processor for estimating reflectivity functions. (a) $\xi = 0.01$ and (b) $\xi = 0.03$.

ization parameter as a filter has been used to improve the estimation of reflectivity functional. This parameter has been selected to control the noise content in the deconvolution result. The simulated of some shape functions were used to study the stability and verification of the proposed algorithm. The algorithm can be used effectively in real time application particularly two and three dimensional images.

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